

Category Theory: Topics for Final Presentation

General Information

The final two lectures of the course (4 December and 11 Decembers) are dedicated to student presentations. These presentations are **individual**. You are expected to pick a topic and give a presentation of 40 minutes about it. The target audience are fellow students following the course. Suggestions for topics can be found below. Note that you are only expected to present the material indicated below, even if reference contains much more. If you would like to present a topic that you picked yourself, please contact me (vanderweide@inf.elte.hu).

The presentations will be graded based on the following aspects.

- Clarity of the presentation
- Correctness
- Completeness

You are expected to present proofs in detail.

Topic 1: Elementary toposes

Motivation Elementary toposes are a categorical structure that find applications in both logic and geometry. In logic, they are used as models of higher-order logic.

Expected Content You are expected to present the following:

- Definition of a subobject classifier
- Definition of elementary topos
- Example: the category of sets is an elementary topos

Literature Up to Example 1.6 (i) in <https://arxiv.org/pdf/1012.5647>

Topic 2: Categories with Families

Motivation Categories with families are used in the study of dependent type theory. They are a categorical structure that represents a model of dependent types. **Note:** knowledge of dependent types is very good to have if you choose this topic.

Expected Content You are expected to present the following:

- Definition of a category with families
- Example: the category of sets induces a category with families
- Relation with dependent type theory

Literature Section 3.1 (up to, but not including 3.1.1) of “Syntax and semantics of dependent types” by Hofmann. Ask me for a copy of the pdf if you are interested.

Topic 3: 2-categories

Motivation The lecture notes contain various digressions that hint at generalisations of category theory.

Expected Content You are expected to present the following:

- Definition of a 2-category
- The 2-category of categories
- Sets and relations form a 2-category

Literature The source <https://arxiv.org/abs/2002.06055> contains the relevant definitions, but much more. In this reference, pick Proposition 2.3.4 as the definition, and Examples 2.3.13 and 2.3.14

Topic 4: symmetric monoidal closed categories

Motivation Symmetric monoidal closed categories are slightly weaker than Cartesian closed categories, but they permit for more examples. One application of symmetric monoidal closed categories is that they are models of linear logic.

Expected Content You are expected to present the following:

- Definition of a symmetric monoidal closed category
- Sets and relations form a symmetric monoidal closed category
- Abelian groups form a symmetric monoidal closed category. This example requires some familiarity with algebra (tensor products of abelian groups)

Literature Section 2 in <https://dl.acm.org/doi/10.1145/3636501.3636956> without Remark 2.2, Example 2.3, and Definition 2.4.

An alternative source is <https://www.sas.rochester.edu/mth/sites/doug-ravenel/otherpapers/malkiewich23.pdf>: Definition 4.1.1 to Theorem 4.1.4.

Note: you only have to present one of these two.